Predicting CPI in Small Areas from Sparse Survey Data

Vladislav Beresovsky [1], Terrance D. Savitsky [1]

[1] U.S. Bureau of Labor Statistics
Office of Survey Methods Research

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Thank you!

Outline

Introduction to Consumer Price Index (CPI)

Modeling price changes (PC) in CBSA

Models evaluation

References

Consumer price Indexes and price changes (PC)

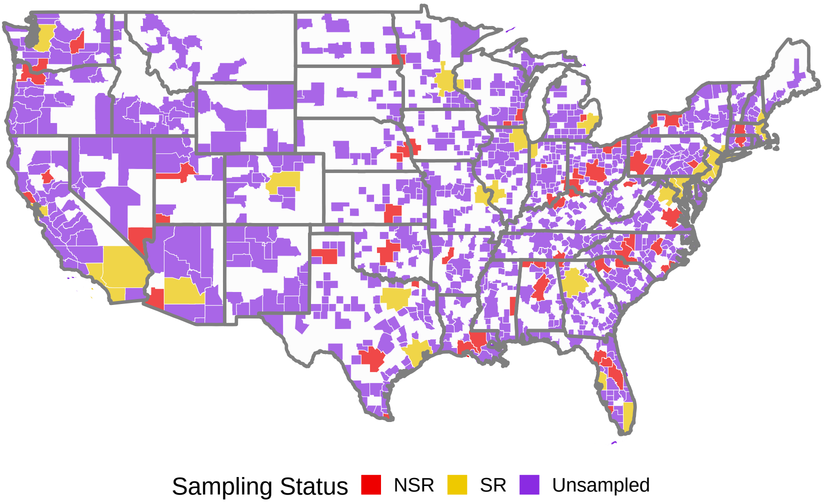
- BLS collects prices for $\approx 100,000$ goods and services paid by urban households of core-based statistical areas (CBSAs), see the BLS Handbook of Methods
- CPI-U population covers 93% of the U.S. population
- The BLS calculates 7,776 basic indexes series $I^c_{a[t]}$ for
  - 32 geographic index areas a
  - 243 commodity items c
  - month t
- The published Indexes are normalized to 100 starting from a base period (originated in 1983 for most items)
- PC over the last p-months. Published PC estimates include monthly ($p = 1$) and annual $p = 12$

$$Y^c_{a[t]} = \frac{I^c_{a[t]}}{I^c_{a[t-p]}} - 1.$$

Geography of CPI sampling and estimation

- Self-representing (SR) CBSA
  - 21 continental large metropolitan CBSA (pop > 2.5M) + AK & HI
  - SR CBSA represent 42% of CPI-U population
  - Estimates are published for all SR CBSA
- Non self-representing (NSR) CBSA
  - 52 NSR CBSA are sampled from the population of ≈ 900 metro- and micropolitan CBSA covering 58% of CPI-U population
  - Data collected in NSR CBSA is used to produce estimates in 9 Census Divisions
- CBSA sample is sparse geographically

Sparse geographic coverage of the CBSA sample



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Estimation goals

- Use cross-sectional data of 12-month fuel price changes
- Use models to smooth out estimates in the 73 sampled CBSA, and to make predictions in 821 unsampled CBSA
- Aggregate estimates in CBSA to State and Division levels
- Validate model-based estimates by comparing with administrative data

Main concepts implemented in our models

1. Fay-Herriot area model at CBSA level (Fay and Herriot, 1979)
2. Bayesian calibration of CBSA predictions to Census Divisions predictions (Savitsky, 2016)
3. Co-modeling of means and variances in small areas (Sugasawa, Tamae, and Kubokawa, 2017), (Savitsky and Gershunskaya, 2023).
4. Global-local (Horseshoe) shrinkage prior for regularized model selection (Carvalho, Polson, and Scott, 2010);
5. Spatial model incorporating dimension reduction and alleviation of confounding (Hughes and Haran, 2013)
6. Multiplicative Gamma shrinkage prior for selecting spatial factors (Bhattacharya and Dunson, 2011)

FH HB area model. Calibration to Census Division

Fay and Herriot (1979) hierarchical model for CBSAs

$$\hat{y}_{i(j)}|y_{i(j)} \overset{\text{ind}}{\sim} N\left(y_{i(j)}, \hat{V}^2_{i(j)}\right),\ j \in 1, \dots, 73$$

$$y_i|\boldsymbol{\beta}, \tau^2_\mu \overset{\text{ind}}{\sim} N\left(\boldsymbol{x_i^T}\boldsymbol{\beta}, \tau^2_\mu\right),\ i \in 1, \dots, 894$$

$\hat{y}_{i(j)}, \hat{V}^2_{i(j)}, y_{i(j)}$ – direct and model-based estimates in sampled CBSA

y_i – model-based predictions in all population CBSA

Calibration of CBSA predictions y_i to Census Divisions

$$\hat{y}_d \overset{\text{ind}}{\sim} N\left(y_d, \hat{V}^2_d\right), d \in 1, \dots, 9$$

$$y_d = (N_d)^{-1} \sum\nolimits_{i \in d} y_i N_i,\ N_d = \sum\nolimits_{i \in d} N_i$$

$\hat{y}_d, \hat{V}^2_d, y_d$ – direct and model-based estimates in Census Divisions

N_i, N_d – CBSA and Division population counts

Co-modeling of variances

Smoothing of direct variance estimates $\hat{V}^2_{i(j)}$, see Savitsky and Gershunskaya (2023)

$$\hat{y}_{i(j)}|y_{i(j)} \overset{\text{ind}}{\sim} N\left(y_{i(j)}, \nu^2_{i(j)}\right), j \in 1, , 73$$

$$\hat{V}^2_{i(j)}|a, \nu^2_{i(j)}, b \overset{\text{ind}}{\sim} G\left(\frac{an^*_{i(j)}}{2}, \frac{an^*_{i(j)}}{2b\nu^2_{i(j)}}\right)$$

$$\nu^2_{i(j)} \overset{\text{ind}}{\sim} IG\left(2, \exp\left(\boldsymbol{z}^{\boldsymbol{T}}_{\boldsymbol{i(j)}}\gamma\right)\right)$$

$\nu^2_{i(j)}$- latent variances in sampled CBSA
$\boldsymbol{z}^{\boldsymbol{T}}_{\boldsymbol{i(j)}}\gamma$- linear modeling of latent variances
$n^*_{i(j)}$ - standardized CBSA sample size
$b \sim G(3, 3)$- uniform bias factor of variance estimation
$a \sim \exp\left(N(0, 1)\right)$- uniform shape factor of variance estimation.

Regularized model selection with horseshoe prior

$p \approx 30$ covariates are used to model 73 sampled CBSA. Model selection is regularized by the choice of prior for regression coefficients.

- No regularization. Multivariate normal prior with LKJ (Lewandowski, Kurowicka, and Joe (2010)) random correlation matrix

  $\boldsymbol{\beta} \sim \text{MVN}\left(0, \Sigma(\eta)\right)$, $\Sigma(\eta) \sim \det\left(E\right)^{(\eta-1)}$
  $\eta \in (1, \infty)$ for positive correlation. We used $\eta = 4$ for moderate correlation.

- Regularization. Horseshoe global-local shrinkage prior (Carvalho, Polson, and Scott, 2010) shrinks parameters estimates to 0, leaving only statistically significant

  $\beta_p \sim N(0, (\sigma_p \sigma_G)^2)$,
  $\sigma_p \sim C^+(0, 1)$, $\sigma_G \sim C^+(0, 1)$

Modeling spatial correlations between CBSA

- No spatial correlations. Independently distributed random effects
  $E\left(y_i|\boldsymbol{\beta}, u_i\right) = \boldsymbol{x_i^T}\boldsymbol{\beta} + u_i,\, u_i \overset{\text{ind}}{\sim} N(0, \tau_u^2),\, i \in 1, \ldots, n$
- Spatial correlations, see Hughes and Haran (2013)
  $E\left(y_i|\boldsymbol{\beta}, \boldsymbol{\beta}_s\right) = \boldsymbol{x_i^T}\boldsymbol{\beta} + \boldsymbol{M}_i\boldsymbol{\beta}_s$

  $\boldsymbol{\beta}_s \sim \mathrm{MVN}(0, \boldsymbol{\Sigma}_s)$ - m-vector of random effects
  $\boldsymbol{\Sigma}_s = (\boldsymbol{M^T}\boldsymbol{Q}\boldsymbol{M})^{-1}$ - $m \times m$ spatial correlation matrix

  $\boldsymbol{A}$ - $n \times n$ adjacency or proximity, e.g., inverse distance matrix

  $\boldsymbol{Q} =$ diag$(\boldsymbol{A}\mathbf{1}) - \boldsymbol{A}$ - $n \times n$ precision matrix

  $\boldsymbol{M}$ - $n \times m$ spatial basis (harmonics), first positive $m \approx 0.10n$ eigenvectors of Moran operator $\boldsymbol{P}^{\perp}\boldsymbol{A}\boldsymbol{P}^{\perp}$
  $\boldsymbol{P}^{\perp}$ - orthogonal projection complement for covariates $\boldsymbol{X}$

Regularized spatial model

In case of factor analysis, Bhattacharya and Dunson (2011) used multiplicative shrinkage prior to regularize variable selection. It allows gradually restricting selection of spatial harmonics with higher indexes h.

We want to

$$\boldsymbol{\beta}_{sh} \sim N(0, (\sigma_h \tau)^2)$$

$$\sigma_h = \prod_{l=1}^{h} \delta_l, \delta_l \sim G(2,1) \text{ local shrinkage parameter}$$

$$\tau \sim C^+(0,1) \text{global shrinkage parameter}$$

ACS and NAICS2 CBSA-level covariates

- American Community Survey (ACS) covariates

  low-income CBSA population $15K-25K (%),
  high-income CBSA population $75+K (%),
  median income value, property value,
  people with Bachelor degree or higher (%), computer users (%),
  young population of 25-31 years old(%),
  urban population (%) and population density.

- Employment distribution (%) by 21 NAICS2 industry codes
- Generated PC basis vectors and retained those explaining up to 95% variability between observations.

Four Models

1. Model 1- FH area model, calibration to Census Divisions, co-modeling means and variances;
2. Model 2- Model 1 + global-local shrinkage prior for regularized covariate selection;
3. Model 3- Model 2 + spatial harmonics;
4. Model 4- Model 3 + multiplicative Gamma shrinkage prior for spatial harmonics.

Models were fit by running CmdStan from RStudio.

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Model fit for 12-month fuel price change

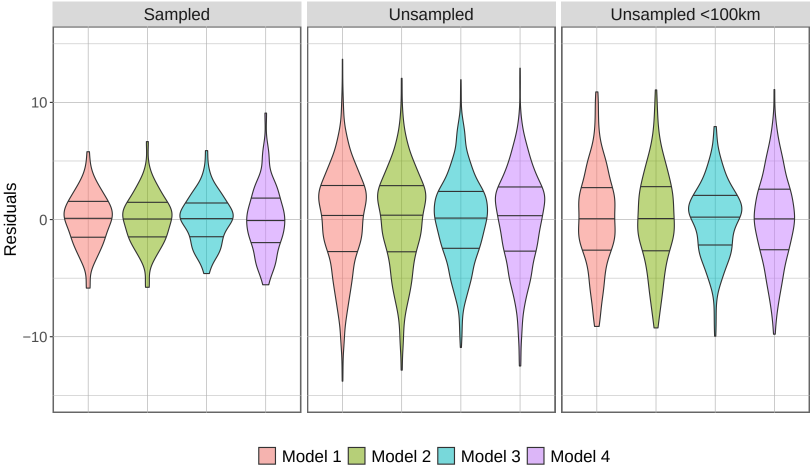
$\hat{Y}_i^{\text{pred}}$- Models predictions in all population CBSA
Y_i^{Admin}- Administrative data in almost all CBSA

Regress Admin on predicted Y: $Y_i^{\text{Admin}} \sim \beta * \hat{Y}_i^{\text{pred}} + \epsilon_i$

Model	RMSE	MAD	β	SD(ϵ_i)	R^2
	73 sampled CBSA				
1	2.26	1.75	0.92	2.23	0.70
2	2.21	1.69	0.93	2.19	0.71
3	2.12	1.65	0.95	2.05	0.74
4	2.93	2.27	0.99	2.83	0.51
	821 unsampled CBSA				
1	5.14	3.99	0.33	4.19	0.11
2	4.62	3.66	0.44	4.14	0.13
3	3.73	2.92	0.75	3.62	0.34
4	5.23	4.19	0.39	4.02	0.18
	235 unsampled CBSA < 100km from the sampled				
1	4.97	3.96	0.36	3.99	0.11
2	4.51	3.64	0.46	3.97	0.12
3	3.09	2.52	0.93	3.05	0.48
4	5.06	4.16	0.46	3.72	0.23

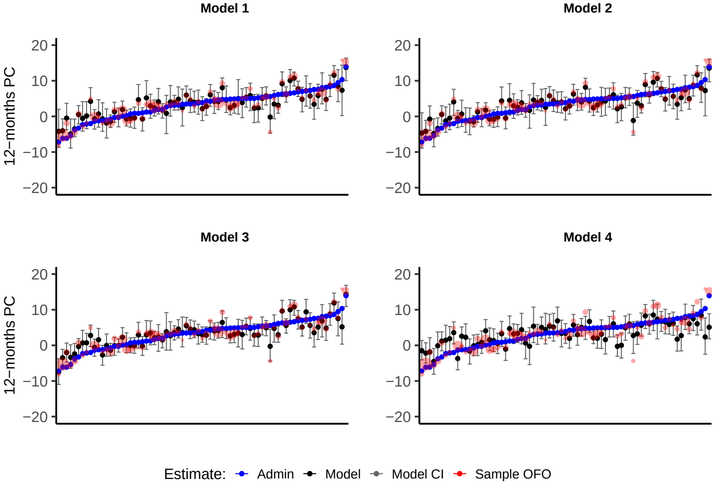
Residuals distribution

Residuals $Y_i^{\text{Admin}} - \hat{Y}_i^{\text{pred}}$ are distributed closer to 0 for Model 3 with spatial correlations.



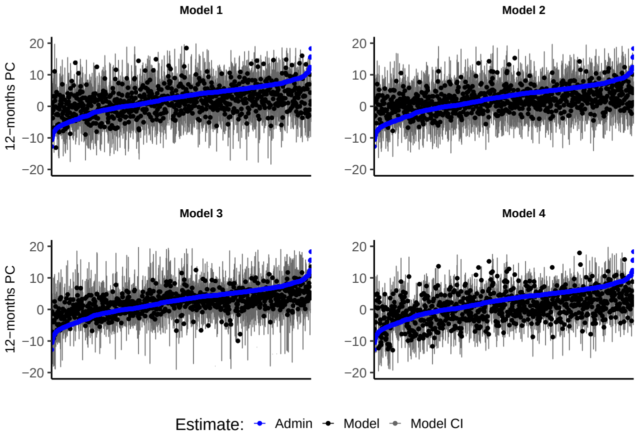
73 sampled CBSA

All models fit Admin data equally well



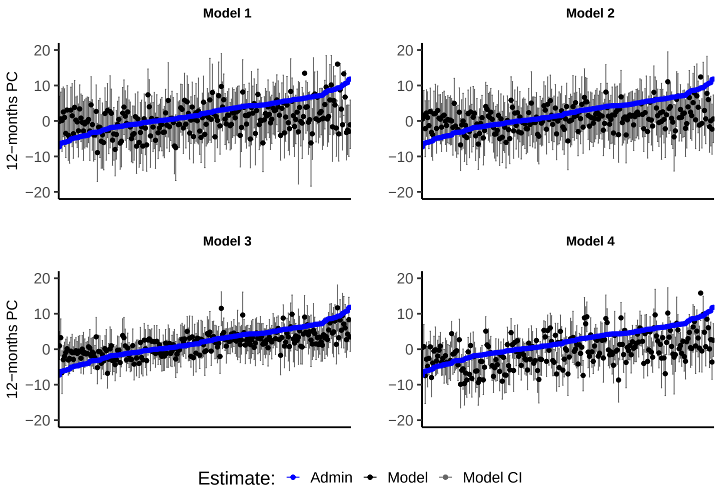
821 unsampled CBSA

Spatial Model 3 fits Admin data better than other

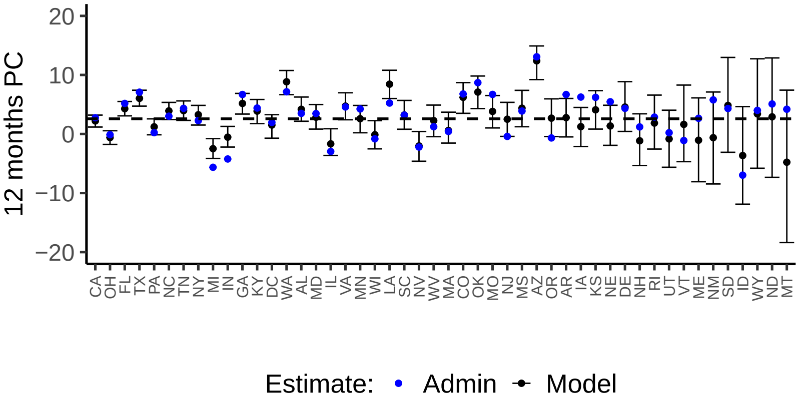


235 unsampled CBSA < 100km from the sampled

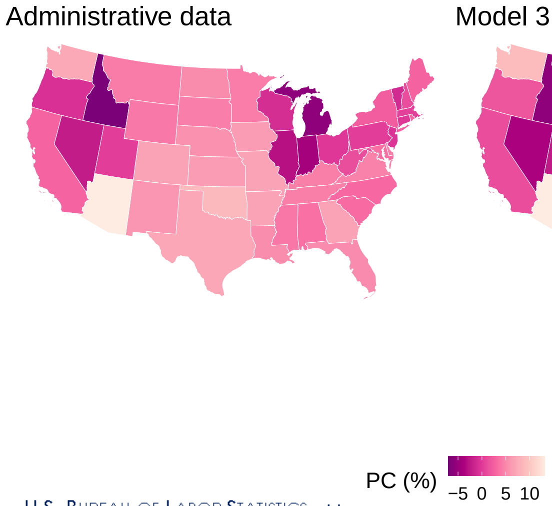
Spatial Model 3 fits Admin data even better



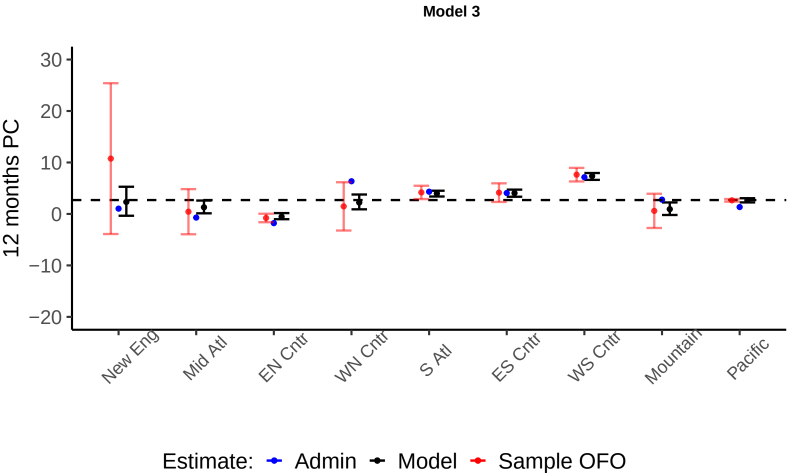
State estimates by spatial Model 3



Spatial Model 3 vs Admin data



Census Divisions estimates



Discussion and next steps

- Accounting for spatial correlations improved model fit to administrative data.
- Fuel price changes across geographies are not effectively explained by ACS and NAICS2 covariates.
- Spatial model can produce aggregated state-level estimates for states with sampled CBSA, or closely located to them. Estimates for distant states without sampled CBSA (MT, ME, NM) have wide confidence intervals.
- We plan fitting price change series using spatio-temporal models.

Contact information

Vladislav Beresovsky
Bureau of Labor Statistics (BLS)
Office of Survey Methods Research (OSMR)

beresovsky.vladislav@bls.gov

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